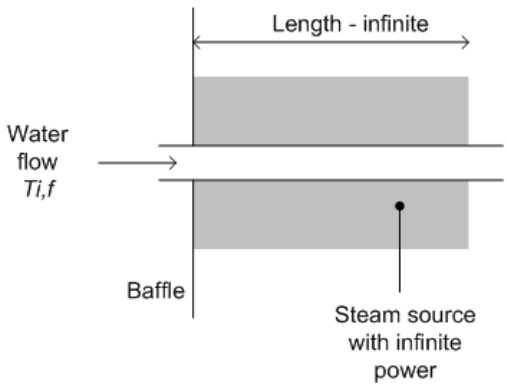
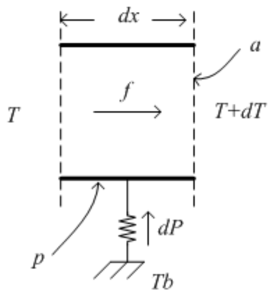
**Condenser Maths**

Consider a pipe containing flowing water surrounded by a steam source of infinite power (or at least much more than the pipe can absorb). The system has a start point where *x=0* and the water temperature *Ti* is known. Water flows at *f.* We seek the equation for the temperature of water in the pipe.




Now consider a short section of that pipe, symbols as shown below.




| Symbol | Meaning |
|---|---|
| T(x) | Temperature at point *x.* |
| dT | Incremental change in temperature across the element |
| P(x) | Power per unit length crossing pipe boundary at point *x.* |
| dP | Power per element dx. |
| p | Perimeter of element |
| f | Fluid flow |
| a | Cross sectional area of element |
| Tb | Boiling point |

$$dP = P(x)dx = sf\,dT \qquad 1.$$

$$\frac{dT}{dx} = \frac{P(x)}{sf} \qquad 2.$$

$$dP = pH(Tb - T(x))dx \qquad 3.$$

As a sanity check, consider a length X over which the temperature is constant T, the area is pX and so:

$$P = pXH(Tb - T) \qquad 4.$$

This seems sensible. Power is proportional to area, H and the temperature difference.

So back to the derivation:

Restating 3.

$$\frac{dP}{dx} = pH(Tb - T(x)) \qquad 5.$$

Substituting 1.

$$\frac{dT}{dx} = \frac{}{sf}(Tb - T(x)) \qquad 6.$$

This has a solution of the form

$$T(x) = A + Be^{\frac{-pHx}{sf}} \qquad 7.$$

At $x = 0$

$$T(0) = T_i \qquad 8.$$

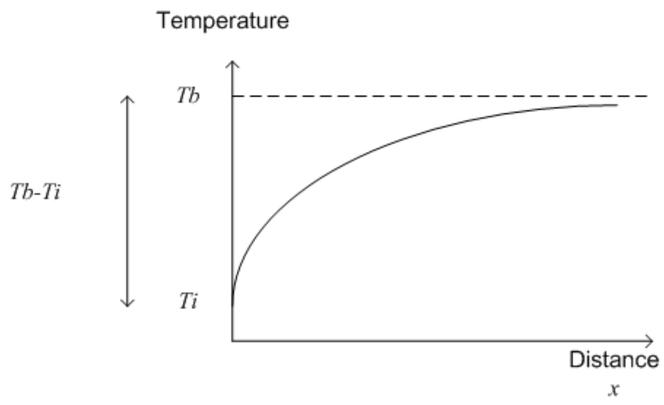
At $x = \infty$

$$T(\infty) = T_B \qquad 9.$$

Therefore

$$T(x) = T_i + (T_B - T_i)(1 - e^{\frac{-pHx}{sf}}) \qquad 10.$$

This looks complicated but it has a simple interpretation. The water starts at *Ti* and absorbs heat from the surroundings at a rate proportional to the difference in temperature between itself and the steam. As it heats it absorbs less as the difference decreases. It asymptotically approaches the boiling point at which it absorbs no more. As the temperature rises exponentially, the heat absorbed falls exponentially. The hot end of the condenser is less effective than the cold end.




The temperature gradient along the pipe is given by

$$\frac{dT}{dx} = \frac{pHx}{sf}(T_B - T_i)e^{\frac{-pHx}{sf}} \qquad 11.$$

In the HTC experiments *x=0* and flow varies, so the exponential is equal to 1.0, so the equation becomes the constant temperature equation 4. :

$$\frac{dT}{dx} = \frac{pHx}{sf}(T_B - T_i) \qquad 12.$$

But when $f \to 0$ this becomes undefined. In Equation 11, the exponential and *1/f* terms have opposite effects, but I can't be bothered working out the limit.

The remaining interesting question is: how low can the flow be when making HTC measurements?

Note that there is nothing in the maths that says the coolant has to be on the inside, versus a Leibig where the fluid surrounds the gas in an annulus.

Engunear, 2015