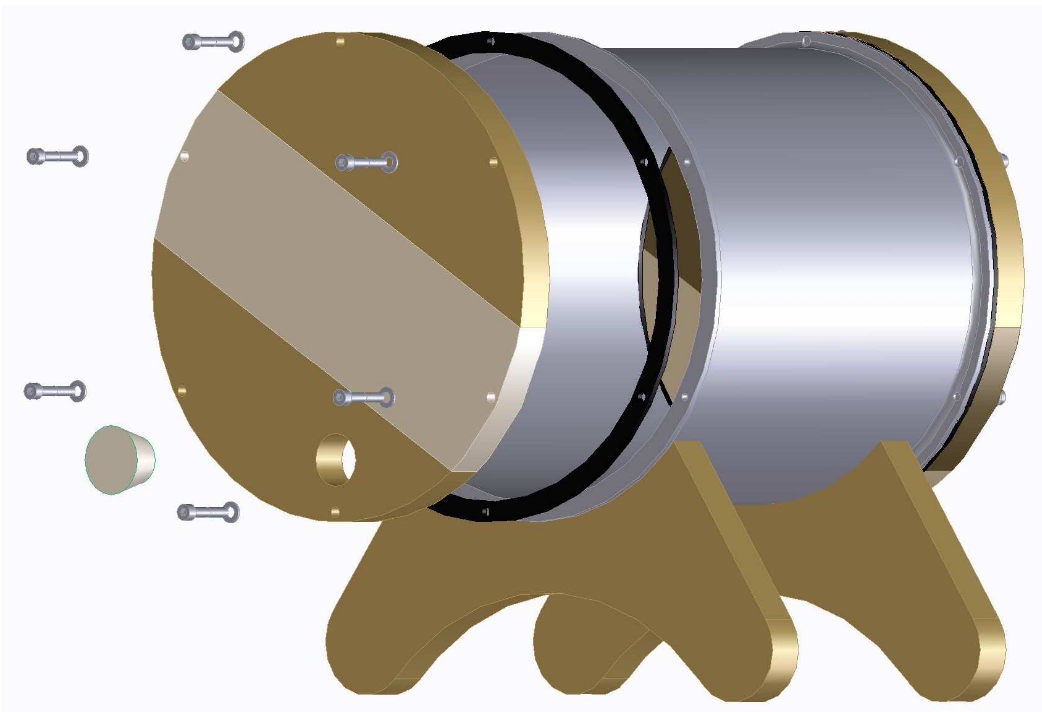
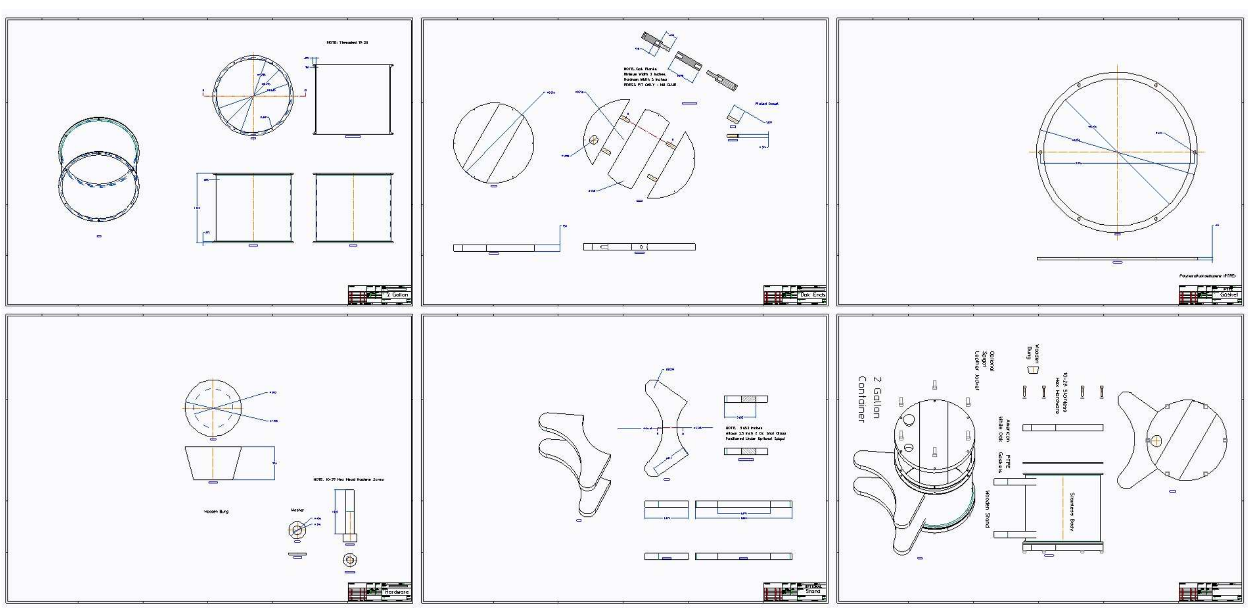
8-Feb-16

How to Age Your 2 Gallons of Scotch The Same as a 53 Gallon Barrel





2 Gallon Stainless Steel Body 8 inches long - American White Oak Ends

Silicone – EPDM or PTFE Gaskets : Stainless Steel Machine Screws : Oak Bung

Patent Pending

Now: Let's Look At The Math

Surface-Area to Volume Ratio – Wooden Ends and Inert Body

A small container with an inert body and wood only for the ends is a simple model. The surface area is the wood that comes in contact with the whiskey, only the ends count. For a container of uniform cross-section, the volume is the area of one end times the length. Therefore, the surface-area to volume ratio is two times the area of one end divided by this same area times the length.

$Sa/V = 2 \times A_i \div A_i \times L_i$

The area cancels and the surface-area to volume ratio is simply 2 divided by the length.

Surface-Area to Volume Ratio of a Small Container with Oak Ends and Inert Body

■ ***Equation 1:*** $Sa/V = 2 \div L_i$

Solving for the length L_i of the small container:

Length of the Small Container

■ ***Equation 2:*** $L_i = 2 \div (Sa/V)$

Where: Sa/V = Surface-Area to Volume Ratio
L_i = Length of Small Container (any shape)

If you know the surface-area to volume ratio of a barrel, you can plug that into Equation 2 and find the length L_i of a small container with oak ends that will have the same surface-area to volume ratio.

You probably already have numbers for the surface-area to volume ratio of different size barrels from tables published on the web. You can use these, just make sure that you pay attention to the dimensions. The *Sa/V* is not a strict ratio. It has a dimension in the denominator. Some tables I have seen use square inches per liter or square inches per gallon. The volume and surface area must be expressed the same units (in^2 to in^3 or cm^2 to cm^3) so you need to convert liters and gallons to cubic inches or cubic centimeters.

If you are happy using the web published tables, you have your length (somewhere around 8 inches for 50 gallons) and you can go ahead and calculate the diameter required to give the volume you are shooting for (assuming you want to use a cylinder).

By the way, you could use any shape for the ends and body It doesn't have to be a cylinder. A 12" x 12" x 8" ID box gives a 5 gallon container which I plan to use with a HDPE body for aging wine.

The Oak Barrel

Although a barrel has a "belly" shape, it is common to use a simple cylinder (Figure 1) to describe the volume and surface area where *R* is the radius, *L* is the length. An additional diagonal dimension, D_g is defined as the diagonal distance from the top center to the lower end of the barrel. The use of D_g will be explained shortly.

Using this simplification does not change the results determined by Equation 1 or 2. If you want to provide a surface-area to volume ratio of a barrel based on integral calculus or actual measurements, these can be used to determine the length L_i that will have the same surface-area to volume ratio, however we will find this complexity is not needed.

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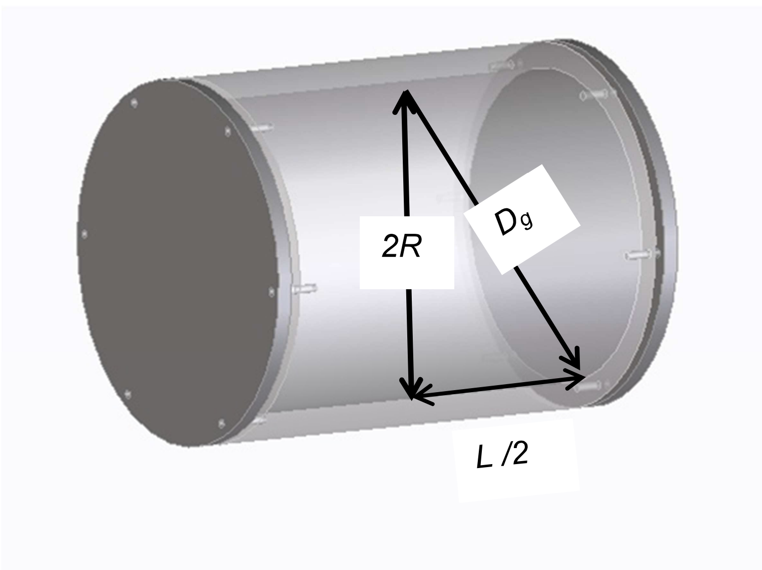

Figure 1: Cylinder Approximation of a Barrel

This surface-area to volume ratio of an oak barrel has been widely discussed ($\boldsymbol{Sa/V}$). The relationship is easy to derive by dividing the area of the cylinder, including the ends, by the volume of the cylinder and has been well published for a wooden barrel as:

Surface-Area to Volume Ratio of Oak Wine Barrel

■ ***Equation 3:*** $\boldsymbol{Sa/V} = 2(\boldsymbol{L} + \boldsymbol{R}) \div (\boldsymbol{L} \times \boldsymbol{R})$

Setting the surface-area to volume ratio of the small container (Equation 1) equal to the surface-area to volume ratio of the barrel (Equation 3), the length L_i of the small container can be calculated.

$$Sa/Vi = 2 \div L_i = Sa/V = 2 \times (L + R) \div (L \times R)$$

The Length of the Small Container

■ **Equation 4** $L_i = (L \times R) \div (L + R)$

Where: L_i = Length of Small Container
L = Length of Large Barrel
R = Radius of Barrel

Shortly we will describe a method to define a barrel and later we will calculate a normalized value and arrive at a true ratio.

We now recall from plane geometry the formula for the volume of a cylinder:

■ **Equation 5:** $V = \pi \times R^2 \times L$

EXAMPLE: If we consider a barrel of length 31.47” and radius 11.13”, we can calculate the volume using Equation 5 as 53 gallons (231 cubic inches per US gallon) and we calculate the length of the small container that will have the same surface-area to volume ratio using Equation 4, as 8.22”.

Defining a Large Barrel

What are the dimensions of a barrel? A cylinder 21 inches in diameter and 35.4 inches has a volume of 53 gallons (Equation 5) and a surface-area to volume ratio of 0.247 (Equation 3). A barrel 23 inches in diameter and 29.5 inches long will also have a volume of 53 gallons but a surface-area to volume ratio of 0.242. Using equation 2 (or equation 4), a small container 8.10 inches long would have the same surface-area to volume ratio as the 21 inch diameter barrel and a small container 8.27 inches would have the same surface-area to volume ratio as the 23 inch diameter barrel.

Which answer is correct? If the surface-area to volume ratio of any barrel is known, then the length of an equivalent small container with wooden ends and an inert body can be calculated exactly using equation 2. If the surface-area to volume ratio of the barrel is obtained from the cylinder model of a barrel, then both answers are correct as approximations applying equation 4. In reality the actual barrel design depends on the cooper. However, for analytical purposes, an ideal barrel has been defined.

Johannes Kepler Optimum Wine Barrel [1]

Johannes Kepler is a famous mathematician who became intrigued with the different wine barrels he encountered in the early 1600’s. Using the cylinder representation for a barrel (Figure 1), Kepler proposed a model for what he considered the optimum wine barrel. Kepler calculated that four times the square of the diagonal ($4Dg^{2}$) should equal three times the square of the length ($3L^2$).

Equation 6: $4D_g^2 = 3L^2$

This allows us to make some interesting calculations:
First, solving for the diagonal length squared (D_g^2):

Equation 7: $D_g^2 = (3 \times L^2) \div 4$

Applying Pythagoras' Theorem for a right triangle, the square of the diagonal is equal to the sum of the square of the other two sides, we can solve for D_g^2:

Equation 8: $Dg^2 = (L \div 2)^2 + (2 \times R)^2$

We now have two equations which are both equal to the diagonal squared $D_g{}^2$ and we can solve for the radius R as a function of the length L.

$$(3 \times L^2) \div 4 = (L \div 2)^2 + (2 \times R)^2$$
$$L^2 \div 2 = 4 \times R^2$$

■ **Equation 9:** $R = L \div \sqrt{8}$

Recalling the volume of a cylinder (Equation 5: $V = \pi \times R^2 \times L$) and substituting the value R from Equation 9), the length L of Kepler's optimum barrel can be determined as a function of the volume V.

■ **Equation 10:** $L = 2 \times \sqrt[3]{V \div \pi}$

Equations 5, 9 and 10 define the volume, radius and length of the optimum wine barrel as defined by Johannes Kepler. This is a theoretical design but turns out to be quite close to barrels being built then and those built today. More importantly, it provides a way to look at the small container with oak ends and draw some useful conclusions.

Volume in Gallons	Volume in Cubic Inches	Kepler[1] Length Inches	Kepler[1] Radius Inches	$\frac{Sa}{V}$ Inches ^{2} to Inches ^{3}	$\frac{Sa}{V}$ Normalized to 53 Gallon Reference Barrel *	*Length In Inches of Small Container Inches*
0.2	46.2	4.90	1.73	1.563	6.42	1.38
1	231	8.38	2.96	0.914	3.76	2.19
2	462	10.56	3.73	0.725	2.98	2.76
5	1155	14.33	5.07	0.534	2.20	3.74
10	2310	18.05	6.38	0.424	1.74	4.72
20	4620	22.74	8.04	0.337	1.38	5.94
30	6930	26.04	9.20	0.294	1.21	6.80
40	9240	28.66	10.13	0.267	1.10	7.48
53	12243	31.47	11.13	0.243	1.00	8.22
59	13629	32.62	11.53	0.235	0.96	8.52
65	15015	33.69	11.91	0.227	0.93	8.80
70	16170	34.53	12.21	0.222	0.91	9.02
100	23100	38.89	13.75	0.197	0.81	10.16

Table 1: Comparison of Surface-Area to Volume Ratios of Different Size Barrels and Small Container Length that will have the same Surface-Area to Volume Ratio

* NOTE: The normalized surface-area to volume ratio is the surface-area to volume ratio of a barrel divided by the surface-area to volume ratio of a 53 gallon reference barrel. This value is a true dimensionless ratio.

For a given volume V, Table 1 shows the calculated values for length L (Equation 10), radius R (Equation 9), surface-area to volume ratio (in ^{2} / in ^{3} - Equation 3), a normalized surface-area to volume ratio (See NOTE*); and the calculated length of a small container that will have the same surface-area to volume ratio L_i (Equation 2 or 4).

Graph 1 plots the normalized surface-area to volume ratio of the all-wooden barrel (Blue) versus the volume. Superimposed on this is the plot of the length of a small container in inches (Red) versus the volume of the equivalent wooden barrel.

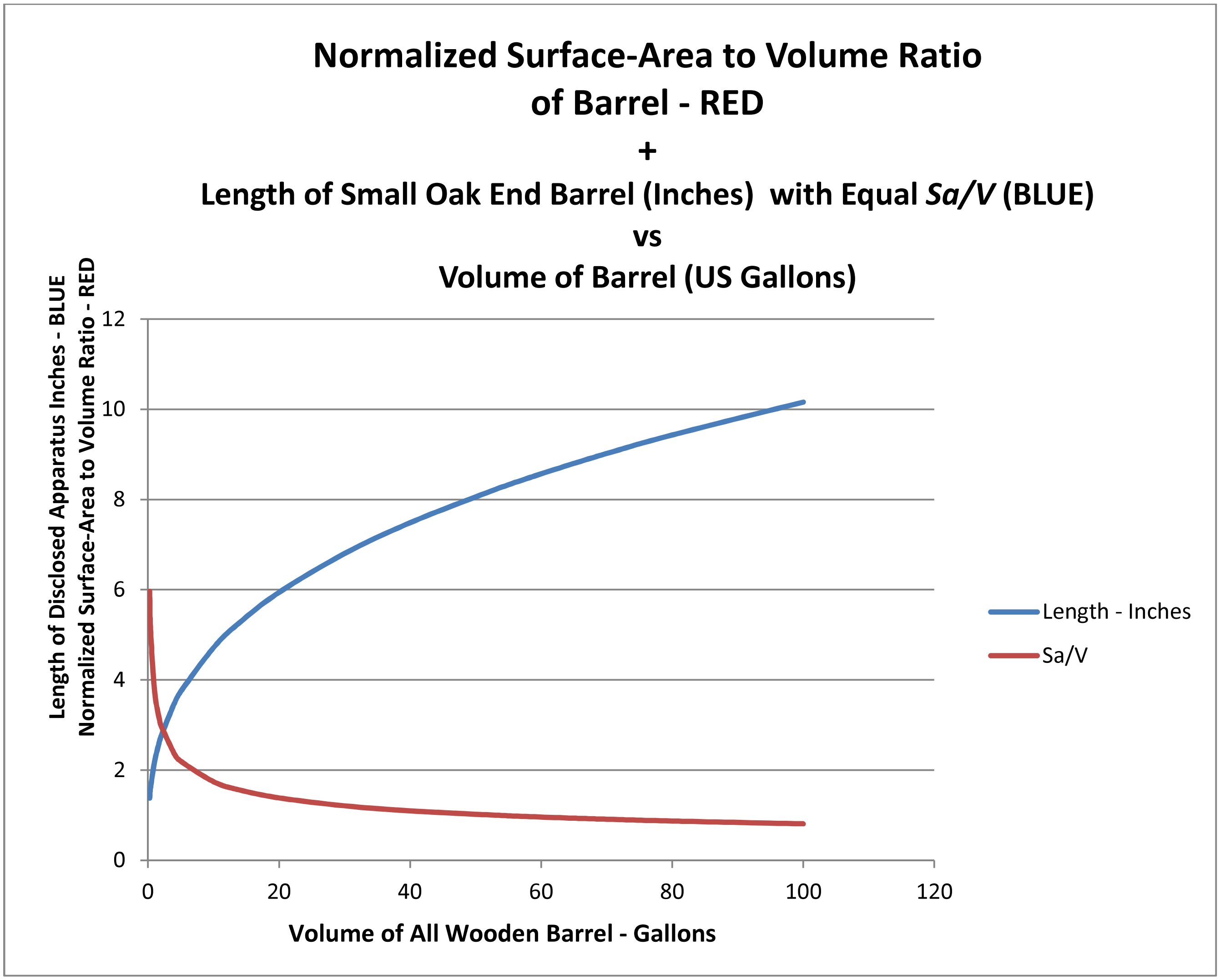

Table 1 and Graph 1 show that for small barrels from 1 gallon to 20 gallons in size, the normalized surface-area to volume ratio decreases rapidly as the volume increases whereas for barrels 20 to 100 gallons in size, as are typically used for commercial production, the Surface-Area to Volume ratios changes less as the volume is increased.

In addition to providing a small container that will age spirits the same as a large barrel, there are several other advantages.

- The inert body is easily cleaned after aging is completed.
- Replace the oak ends and gaskets, and the container is equivalent-to-new.
- The initial cost of the container can be amortized over many batches of whiskey.
- The wood used for aging can be easily changed.
- The small size containers can be stored in a refrigerator outfitted with a temperature and possibly a humidity controller to even simulate the environment of the commercial facility.

The important thing is that for any barrel for which you can measure or estimate the surface-area to volume ratio, there is a length that can be calculated such that a container with wood ends and inert body can be built that has the same surface-area to volume ratio.

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